Appendix

A1. Nominal per-capita GDP

To estimate nominal per-capita GDP for the 19th century, we ran a regression for the 1900–1947 period, with the log of the nominal per-capita output (from Haddad, 1978) as the dependent variable. As independent variables the logs of an arithmetic mean of exports and imports of goods in national currency (which we denominate as "foreign trade index"), the arithmetic mean of central government revenues and expenditures (which we denominate as "government budget"), and a money-supply index, all expressed in per-capita terms.

Haddad's nominal output is available only from 1908 onwards, so to obtain a nominal GDP series for the entire 1900–1947 period, we multiplied Haddad's real output series by the Catão (1992) wholesale price index and linked the results to Haddad's nominal output in 1908. The variables used in the model are expressed in current prices with base year 1900 = 100.

Our estimates of the Brazilian population for 1820 to 1915 are from Mortara (1941) and for 1916–1947 from Ipeadata (2022). We use Mortara's data through 1915 taking into consideration the author's observation that the 1900 Census, used by Ipeadata, underestimated the population in residence.

For the period 1821–1913, we use the exports and imports series of Absell and Tena-Junguito (2018, online appendix) converted from British Pounds to *Mil-réis* by the official exchange rate in Brazil (1917, p. 243). We used the Menezes' (2010) series of Brazil's exports and imports chained to Absell and Tena-Junguito for 1820. For the period 1914–1947, we used the official series of export and import in local currency of IBGE (1990, pp. 570–571). The official series was linked to the Absell and Tena-Junguito series in 1913.

The series of central government's revenues and expenditures are the revised series of Carrara (2022) for 1820–1898. In a personal communication, Carrara kindly provided us with the data for 1899 and 1900. By official decree of 8 October 1828, the fiscal year began on 1 July and ran to 20 June of the following year. The equivalence between the calendar and the fiscal year was reestablished in 1888. We took the arithmetic mean between two consecutive fiscal years to make the calendar and the fiscal year coincide. The series of central government's revenues and expenditures for the 1901–1947 period is from IBGE (1990, pp. 533–539).

The monetary series is from Peláez and Suzigan (1981), as reproduced without moving average adjustments in IBGE (1990, pp. 533–539). This money supply series was constructed as follows: currency issued from 1820 to 1838, M1 plus demand and term deposits at Banco do Brasil from 1839 to 1851, and the M2 monetary aggregate from 1852 to 1947. We joined the currency issued series to that of M1 plus demand and term deposits at Banco do Brasil in 1839.

In Table A1, we show the descriptive statistics of the variables expressed in annual percentage change (log differences). We divided the sample into three parts: 1821–1870, the time of Independence until the end of the Paraguayan War; 1871–1900, after the War until the end of the 19th century; and 1900–1947, which was the sample used in the regression of nominal output per capita.

Table A1. Descriptive statistics of the variables - annual percentage changes*

Variables	Mean	Median	Standard deviation	Coefficient of variation	Skewness	Kurtosis
1821–1870						
Foreign Trade Index	2.86	2.34	15.01	5.26	-0.10	0.52
Government Budget	3.34	3.02	15.43	4.62	-1.34	6.99
Money Supply	5.16	3.20	9.81	1.90	1.60	4.62
1871–1900						
Foreign Trade Index	2.25	1.72	15.04	6.69	0.91	1.79
Government Budget	1.86	0.97	13.01	7.01	-1.61	7.73
Money Supply	2.14	-0.12	18.43	8.62	2.26	6.83
1901–1947						
Haddad's output	5.93	6.20	12.62	2.13	-0.38	-0.33
Foreign Trade Index	5.34	7.00	18.04	3.38	-0.38	0.58
Government Budget	5.65	7.50	12.09	2.14	-0.21	-0.40
Money Supply	6.87	6.85	10.87	1.58	0.37	0.02

Note: *percentage changes measured in log differences.

In Table A2, we apply an augmented Dickey–Fuller test (ADF) to the series to test the null hypothesis that a unit root is present in the time series samples. The high p-values obtained confirm that all series have a unit root.

Table A2. Augmented ADF test, variables in levels in per capita terms: 1900-1947

Variables	Constant			Constant and trend		
	lags	t-stat	p-value*	lags	t-stat	p-value*
Log of Nominal Output	0	1.7512	0.9996	0	-1.7775	0.6997
Log of Foreign Trade Index	0	1.1072	0.9970	0	-1.5359	0.8027
Log of Gov. Budget	0	1.6416	0.9994	0	-1.4990	0.8160
Log of Money Supply	1	0.5219	0.9857	1	-2.4503	0.3502

Null hypothesis: variable has a unit root; Lag length selected based in SIC (maxlag = 9).

Note: MacKinnon (1996) p-values.

In Table A3, we present cointegration tests. The Engle-Granger test consists of a unit root test on the residuals of the cointegrant regression shown in Table A4. The Engel-Granger test rejects the null hypothesis of a unit root in the residuals (in both variants of no intercept and with intercept) and therefore indicates the possible presence of cointegration among the variables. Johansen's test rejects the null hypothesis of no cointegrating equations in the no-intercept-no-trend specification. In the estimated equation in Table A4, the intercept is statistically insignificant consistent with the Johansen test in no-intercept-no-trend specification.

Table A3. Cointegration tests of Engle-Granger and Johansen: 1900-1947***

Engle-Granger (null: no coint.)			Johansen (null hypothesis: no cointegration)				
	No intercept or trend	Intercept and no trend	Cointegration Equations	No intercept or trend		Intercept and no trend	
				Trace	L-max	Trace	L-max
Lags	0	0		1	1	1	1
tau-stat.	-5.8003 (0.0003)**	-5.8435 (0.0013)**	**None**	40.958 (0.0416)*	18.868 (0.2216)	45.766 (0.2227)	18.893 (0.5007)
z-stat.	-39.572 (0.0004)**	-40.354 (0.0008)**	**At most 1**	0.2500 (0.0920)	13.232 (0.2132)	26.873 (0.2953)	14.664 (0.4032)
			At most 2	0.1223 (0.1775)	6.0016 (0.3495)	12.210 (0.4302)	8.7718 (0.4587)
			At most 3	0.0602 (0.1077)	2.8561 (0.1077)	3.4379 (0.5022)	3.4379 (0.5022)

Notes: * Denotes rejection of the hypothesis at the 5% level and ** at the 1% level. *** p-values in brackets, MacKinnon (1996) p-values in the Engle-Granger test and MacKinnon, Haug and Michelis (1999) in the Johansen test.

Results of the nominal per-capita output regression are presented in Table A4.

Table A4. Nominal per-capita output regression: 1900–1947

	Coefficient	std. error	t-ratio	p-value	
Dependent variable: log of nominal GDP			regression standard error = 0.0621		
HAC standard errors & covariance					
(Bartlett kernel, Newey-West automatic bandwidth = 4.6293, lag length = 3)					
Constant	-0.175493	0.130513	-1.345	0.1856	
Foreign Trade Index (log)	0.461783	0.069254	6.668	0.0000	***
Government budget (log)	0.179754	0.094806	1.896	0.0645	*
Money supply (log)	0.387892	0.066698	5.816	0.0000	***
n. = 48 R^2 = 0.9953 R^2adj. = 0.9950 F(4,43) = 3096 Durbin-Watson = 1.69					

Note: * Indicates significance at 10%, ** at 5%, and *** at 1%.

As expected in a cointegrating regression, R^2 is high and the regression coefficients are all significant. They add up to just greater than one (1.0294) and a restriction that they must add up to one (1.00) is not rejected by a Wald F test with a p-value of 30.73% (F-statistic = 1.0667, degree of freedom = 1,44). Using the coefficients of the regression in Table A4, we distribute them by the pro rata method so that the variables weights add up to exactly 1. Using the calculated weights, we construct a Laspeyres index for nominal per-capita product with 1900 as the base year ranging from 1820 to 1900:

$$Y^n_t = 100 \times \left(0.448582 \times \hat{FT}_t + 0.174615 \times \hat{GB}_t + 0.376803 \times \hat{M}_t \right)$$

where the hats on top of the variables indicate the (gross) percentage change between year t and year 1900; Y^n_t is the nominal output; FT_t is the foreign trade index obtained by the arithmetic mean of exports and imports; GB_t is the "government budget" obtained by the arithmetic mean of government expenditures and revenues; and M_t refers to a series of money supply. All variables are in per-capita terms. In the interval between 1890 and 1892, we excluded the money supply from the construction of the nominal per-capita product index to avoid the distorting effects of the huge monetary expansion that occurred in the transition from monarchy to republic.

A2. Output deflator

The most rigorous Brazilian price index for the latter part of the 19th century is Catão (1992). This author presented "a new wholesale price index based on a much broader basket of goods and on a macroeconomically representative weighting system derived from the first national production census in 1919" (Catão, p. 519). Catão's main source of data was Brazil's most important newspaper at the time—the Jornal do Commercio—where he obtained price estimates for 30 products: beans, beer, Brazilian brandy (*aguardente*), butter, candles, cement, cod fish, coffee, corn, dried meat, lard, ham, Italian pasta, linseed oil, kerosene, manioc flour, matches, olive oil, pinewood, rice, salt, sugar, tallow, tar, tea, tobacco, turpentine, vinegar, wheat flour, and wine.

Although the price quotations are for the city of Rio de Janeiro, Catão believed there were strong reasons to accept such prices as representative at the national level. First, he argued that Rio was the most important Brazilian economic centre in the 19th century, supplanted by São Paulo only after the second decade of the 20th century. Second, "a rough comparison between the indices of Lobo et al. (1971) for Rio and those of Mattoso (1978) for Salvador and Eisenberg (1974) for Recife showed that price trends were very similar across these state capitals" (Catão, p. 521). Third, some commodities sold in Rio were, in fact, imported from other Brazilian regions. The only problem with Catão's index was that it is available only from 1870, hence it needed to be linked to others to cover the 19th century.

Lobo et al. (1971) is a widely used source for domestic price indices in the 19th century. She estimated three price indices based on nine food products in the city of Rio de Janeiro between 1820 and 1930:[1] sugar, rice, cod, coffee, dry meat, wheat flour, manioc flour, beans, and butter. Each index contains the same goods differing only in terms of the weights: the first index has 1856 as base-year and its weights are inferred from information in Diários da Companhia de Luz Stearica on the amounts (in *Mil-réis*) spent on food by workers, enslaved people, and settlers. The second index has 1919 as its base-year and the weights were borrowed from Affonseca (1919), who built a cost-of-living index for Rio de Janeiro based on his own domestic budget, a household composed of seven individuals. The third weight structure has 1949 as base-year, and is derived from the Getúlio Vargas Foundation consumer price index.

Buescu (1973, p. 233) presented an alternative price index, with a thorough investigation of price changes from 1826 to 1887. From 1826 to 1880, he determined his index mostly through a detailed price search of classified ads in Brazil's main newspaper of the time, the Jornal do Commercio. For 1880 to 1887, his sources were the annual Reports of the Finance Ministry and the yearly Retrospects of Jornal do Commercio. For this last period, he collected yearly price changes whereas for 1826 to 1880 his observations are for selected years (1826, 1830, 1835, 1838, 1842, 1847, 1850, 1853, 1856, 1862, and 1870). The number and

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[1] Due to lack of data, in some years, Lobo et al. (1971) considered the prices of less than nine products.

nature of products in his index changed along the period. On average, he collected prices for 17 products in 1826–1838; 90 in 1838–1850; 44 in 1850–1870; 24 in 1870–1875; 12 in 1875–1880; and 50, in 1880–1887. One weakness of his index is the lack of a weighting structure, as pointed out by Haddad and Versiani (1990, p. 135), but he attempted to deal with this deficiency by indirectly weighting each product by the number of times it appears in his samples.

Buescu's index for Brazil starts in 1826 while his index for Rio de Janeiro is from 1772 to 1819. It was thus necessary to link the two indices. For this, we extended from 1819 to 1824 the inflation rate (2.5% per year) that Buescu conjectured for Rio between 1807 and 1819. For 1825 and 1826, we used the rate of 7% per year that Buescu estimated for Brazil's inflation between 1826 and 1830.

In each chapter of his book, Buescu compares his results with those of other price indexes for the respective period. Of particular interest are the comparisons he makes with the price indexes of Lobo. In general, he finds his index to show lower inflation rates than Lobo's. He attributes this to the fact that Lobo's indexes are restricted to a limited number of food products, the prices of which may have risen by more than the tradable products included in his index. The larger discrepancy is in the 1850–1870 period, for which his accumulated inflation rate is 84% whereas Lobo's vary from 274% to 308%, depending on the weighting structure. This large discrepancy does not show up in Mattoso's (1978) food price index for Bahia, which according to Buescu (p. 176) rose by 90% in the period.

Figure A1 displays the price indices of Buescu, Catão and Lobo from 1820 to 1913 with base year 1880 = 100. From 1820 to 1845, the Lobo price index (1919 weighting) shows a yearly inflation rate of 4.4%; and 4.8% yearly between 1845 to 1870. Between 1820 and 1845, the Buescu price index shows a yearly inflation rate of 2.6%; and 2.5% yearly between 1845 and 1870.

Figure A1 – Price indexes of Buescu, Catão and Lobo, 1820-1913 (1870 = 100)
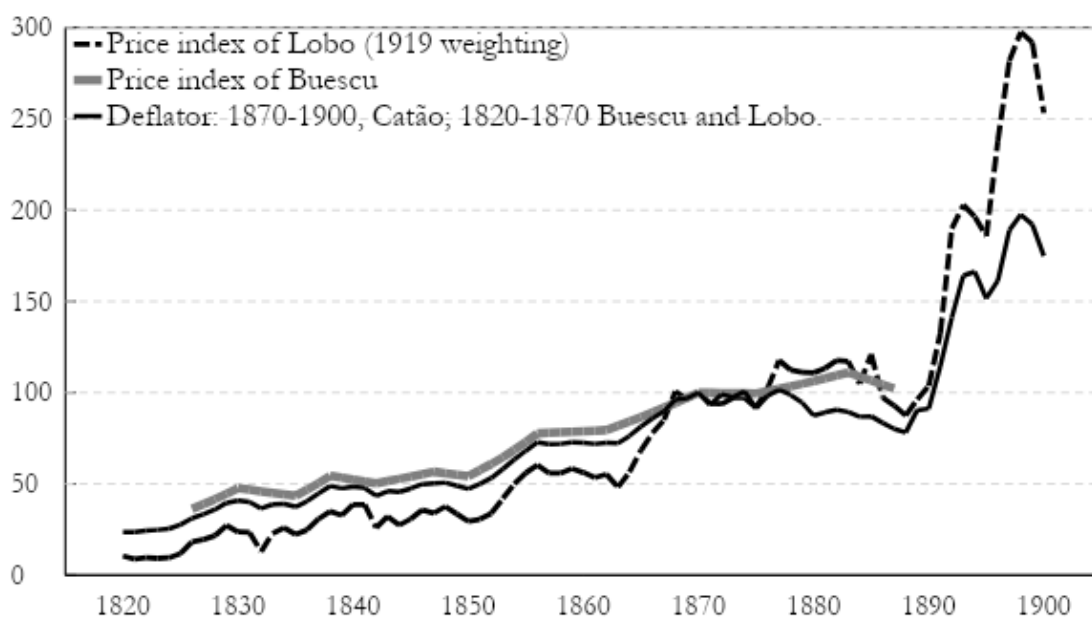
Source: Buescu (1973), Catão (1992) and Lobo et al. (1971).

Comparing the indices for 1870–1887, a period for which the three indices have data, the Buescu price index shows a yearly inflation rate of 0.1%, the Lobo index -0.4% and the Catão index -1.3%. The Catão index exhibits a stronger deflation in the 1870–1888 period that is not fully captured by the other indices. This fact plays a key role in explaining the paradoxical result obtained by Goldsmith of a negative trend growth of Brazilian real per-capita output between 1869 and 1900.

The Catão wholesale price index is available from 1870 to 1913, so we used it as the GDP deflator from 1870 to 1900. One could use the Lobo index to extend the Catão index backward, but this is a food-cost index that relies only on the prices of nine or less commodities. The Buescu price index, on the other hand, does not have a weighting structure but is composed of a larger sample of goods.

We adopt a compromise solution of combining the price indices of Buescu and Lobo—the latter with weights of 1919—to extend Catão's index from 1870 backwards. To do this we first regress the log first differences of the Catão price index on the log first differences of the price indices of Buescu and Lobo for the 1871–1887 period. We impose the restriction that the sum of the two coefficients is equal to one to obtain appropriate weights. We also run the model imposing a zero constant to ensure consistency of the resulting index.

Table A5. Deflator regression: 1871–1887

Regression with the restriction: sum of coefficients = 1					
	coefficient	std. error	t-ratio	p-value	
Δlog of Buescu Price Index	0.716484	0.125696	2.256	0.0385	**
Δlog of Lobo Price Index	0.283516	0.125696	5.700	3.28e-05	***

Thus, our deflator for the 1870–1900 period is the wholesale price index of Catão (1992). For the 1820–1870 period, our deflator is a Laspeyres index, based in 1870, built on the price indices of Buescu (1973) and Lobo et al. (1971)—the latter with 1919 weights—linked to Catão's wholesale price index in 1870:

$$P_t = 100 \times \left(0.716484 \times \hat{B}_t + 0.283516 \times \hat{L}_t \right)$$

where the hats on top of the variables indicates the (gross) percentage change between year t and year 1870; P_t is our proposed output deflator, B_t is the Buescu price index and L_t is the Lobo price index. This Laspeyres index is then spliced into the Catão index in 1870 to extend it back to 1820. This is the price index that we use as a deflator of nominal per-capita GDP in the 19th century.

Goldsmith constructed his deflator based on the average of four price indices: Buescu (1973), Lobo et al. (1971), Onody (1960), and Vieira (1947). We did not consider either Onody's or Vieira's price indices because they suffer from serious shortcomings. Onody's price index is made of 18 imported goods retrieved from the government's customs tariff schedules (for some years the number of goods is reduced to 10). It is available only for selected years scattered through the 19th century (1829, 1834, 1844, 1857, 1860, 1874, 1881, 1887, 1896, 1900). According to Versiani (2023), another major weakness of Onody's price index is that the prices in the government tariff schedules were updated with lags in relation to market prices, in addition to frequent tariff reforms, which made tracking prices difficult. Moreover, the construction of official prices was heavily influenced by importers' lobbies and, in the last decades of the century, by protectionist interests. That is, the relationship between official and market prices was precarious.

Vieira's price aggregate is also problematic. It consists of a weighted mean of yearly unit values (total exports in *Mil-réis* divided by export volumes) of Brazil's nine most important export commodities, from 1821 to 1940. As such it cannot be classified as a true price index, since it is not made from homogeneous products and suffers from a quality bias. Moreover, the underlying official export prices were challenged by Absell and Tena-Junguito (2018) with dramatic changes in their estimated values.

A3. Real per-capita output: structural break tests

Searching for possible multiple breakpoints in the series, we apply Bai-Perron (2003) tests in Table A6. The "Sequential" Bai-Perron test begins testing the null hypothesis of zero versus one breakpoint (0 vs. 1), if the null is not rejected the next hypothesis tested is 1 vs. 2, and so on up to the maximum number of breaks allowed. Already at the beginning, the test does not reject the null of 0 vs. 1 breakpoint at a significance of 5%, and the Scaled F-statistic, 4.76, is smaller than the 5% critical value of 11.47.

Table A6. Multiple breakpoint tests, sample = 1820-1900, n = 81

Bai-Perron Test: sequential (L+1 breaks vs. L)			Bai-Perron Test: Global L breaks vs. none			Information criteria***	
Break Test	Scaled F-statistic	Critical Value**	Breaks	Scaled F-statistic	Critical Value**	Schwarz Criterion	LWZ Criterion
0 vs. 1	4.7643	11.4700	0			-4.4870	-4.4052
			1	4.7643	11.47	-4.4528	-4.2470
			2	9.0160	9.75	-4.3617	-4.0304
			3	9.4533*	8.36	-4.2867	-3.8282
			4	6.6300	7.19	-4.1560	-3.5686
			5	13.526*	5.85	-4.0237	-3.3053

Break test options: trimming 0.15, Max. breaks 5, Sig. level 0.05; HAC covariances (Prewhitening lags = 1, Quadratic-Spectral kernel, Andrews bandwidth); allow heterogeneous error distributions across breaks.

Notes: * Significant at the 0.05 level; ** Bai-Perron (2003) critical values; *** minimum information criterion values displayed with shading.

Likewise, we perform a Bai-Perron of L globally optimised breaks against the null of no structural breaks (L vs. 0 breaks). The test identifies at 5% of significance two possible sets of breaks. A sequence of three breaks: 1847, 1864 and 1878; and another sequence of five breaks: 1832, 1844, 1857, 1870 and 1889. The five break-points hypothesis has the biggest F-scaled statistic of Bai-Perron and could be chosen. To test the two variants of the Bai-Perron test, we use the Schwarz and the Liu-Wu-Zidek information criteria to select the number of breaks. In Table A5, both the Schwarz and the LWZ information criteria select zero breaks (at 5% of significance) as done by the Bai-Perron sequential test.

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